HW5 Problem A2

Design a causal, FIR bandpass filter with w1 = 0.3pi, w2 = 0.6pi.

Use a hamming window.

The equations for the ideal impulse response are::

H\_bp(n) =[ 1/(pi\*n) ] \* [ sin(0.5\*pi\*n) – sin(0.3\*pi\*n)] ( for the type 1 and 2 filters)

And

H\_bp(n) = [1/(pi\*n] \* [cos(0.5\*pi\*n) – cos(0.3\*pi\*n)] (for the type 3 and 4 filters)

I generated even and odd impulse responses based on the above equations, truncated to 7 and 8 samples, and multiplied by the 7/8 sample hamming window

Here are the filter coefficients obtained:

Type 1: -0.0076 -0.0759 0.0348 0.3000 0.0348 -0.0759 -0.0076

Type 2: 0.0034 -0.0550 -0.0925 0.2157 0.2157 -0.0925 -0.0550 0.0034

Type 3: -0.0149 0.0247 0.2198 0 -0.2198 -0.0247 0.0149

Type 4: -0.0141 -0.0228 0.1510 0.1842 -0.1842 -0.1510 0.0228 0.0141

Below is the frequency response plot for the filter. Interestingly, the hamming window reduced the peak gain of the filter to about -5dB.



On a hunch, I decided to divide each impulse response by the ‘mean of the hamming window (0.4743 for the odd filter and 0.4825 for the even filter) which brought the peak gain much closer to 0dB for each filter.



Which brings the frequency response much closer to 0dB in the passband.

Analysis:

A few interesting things to note: The type 3 filter appears to have the best stopband response, particularly near the frequencies 0 and pi. The other filter types (1, 2, and 4) have frequency responses above 0.1 at frequencies 0, pi, or both. Type 1 has the worst stopband performance.

An unexpected result was that the hamming window screwed up the gain in the passband, and required adjustment using the mean of the window to get back to 0dB.

Source Code

%M = 7 or 8 (filter length odd or even)

omega\_1 = 0.3\*pi;

omega\_2 = 0.6\*pi;

n\_inf\_odd = -2000:2000; % really, really long n (centered at origin)

n\_inf\_odd\_center = 2001;

n\_odd\_inds = (-3:1:3) + n\_inf\_odd\_center

n\_inf\_even = -1999.5:1:1999.5; % really, really long n even\_length

n\_inf\_even\_center = 2000.5; %index of center

n\_even\_inds = (-3.5:1:3.5) + n\_inf\_even\_center

%filter 1 : Type 1 (odd and symmetric)

h\_ideal\_type\_1 = (1./(pi \* n\_inf\_odd)) .\* (sin(omega\_2 \* n\_inf\_odd) - sin(omega\_1 \* n\_inf\_odd));

h\_ideal\_type\_1(n\_inf\_odd\_center) = omega\_2/pi - omega\_1/pi;

figure; freqz(h\_ideal\_type\_1)

figure; freqz(h\_ideal\_type\_1 .\* hamming(length(h\_ideal\_type\_1))');

h\_ideal\_type\_2 = (1./(pi \* n\_inf\_even)) .\* (sin(omega\_2 \* n\_inf\_even) - sin(omega\_1 \* n\_inf\_even));

figure; freqz(h\_ideal\_type\_2)

h\_ideal\_type\_3 = (1./(pi\* n\_inf\_odd)) .\* (cos(omega\_2 \* n\_inf\_odd) - cos(omega\_1 \* n\_inf\_odd));

h\_ideal\_type\_3(n\_inf\_odd\_center) = 0;

h\_ideal\_type\_4 = (1./(pi\* n\_inf\_even)) .\* (cos(omega\_2 \* n\_inf\_even) - cos(omega\_1 \* n\_inf\_even));

figure; freqz(h\_ideal\_type\_4)

M\_odd = 6; M\_even = 7;

h\_type1\_rect = h\_ideal\_type\_1(n\_odd\_inds);

h\_type1\_hamming = h\_type1\_rect.\*hamming(M\_odd+1)';

h\_type2\_rect = h\_ideal\_type\_2(n\_even\_inds);

h\_type2\_hamming = h\_type2\_rect.\*hamming(M\_even+1)';

h\_type3\_rect = h\_ideal\_type\_3(n\_odd\_inds);

h\_type3\_hamming = h\_type3\_rect.\*hamming(M\_odd+1)';

h\_type4\_rect = h\_ideal\_type\_4(n\_even\_inds);

h\_type4\_hamming = h\_type4\_rect.\*hamming(M\_even+1)';

% plots.

[H\_rect(:,1),F] = freqz(h\_type1\_rect);

[H\_rect(:,2),F] = freqz(h\_type2\_rect);

[H\_rect(:,3),F] = freqz(h\_type3\_rect);

[H\_rect(:,4),F] = freqz(h\_type4\_rect);

[H\_hamm(:,1),F] = freqz(h\_type1\_hamming);

[H\_hamm(:,2),F] = freqz(h\_type2\_hamming);

[H\_hamm(:,3),F] = freqz(h\_type3\_hamming);

[H\_hamm(:,4),F] = freqz(h\_type4\_hamming);

figure; semilogy(F/pi,abs(H\_hamm));

title('Hamming window on filter lengths 7 and 8');

legend('type 1','type 2','type 3','type 4');

xlabel('Digital Frequency / pi');

ylabel('Magnitude of H');

grid on;

figure; semilogy(F/pi,abs(H\_rect));

title('Rectangular window on filter lengths 7 and 8');

legend('type 1','type 2','type 3','type 4');

xlabel('Digital Frequency / pi');

ylabel('Magnitude of H');

grid on;